Campaign Finance and Welfare When Contributions Are Spent on Mobilizing Voters.

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Abstract

We build a political competition model to analyze the welfare effect of campaign finance policies in a context where parties spend campaign contributions on mobilizing voters—rather than on advertising, as is usually done in this literature. This modification results in key consequences for the welfare evaluation of campaign finance policies. Additionally, we measure the social cost of contributions in terms of the quality lost on public works delivered by contributors. We find that subsidizing campaigns with public funds and simultaneously banning contributions is welfare-improving for citizens only if the parties' mobilization technology is not especially productive. Combining non-matching subsidies with limits on contributions is Pareto improving under same technological conditions. Imposing a contribution lump-sum tax, while simultaneously investing these revenues on public projects is welfare-improving for citizens, and combining this policy with a limit on contributions is Pareto improving. These tax results hold regardless of parties' mobilization productivity level.

Keywords: campaign finance, voters mobilization, welfare, efficiency

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1 Introduction.

Campaign contributions entail both benefits and costs for citizens. Benefits are related to the need for money to bring candidates and citizens closer to each other, as well as to mobilize the latter to express their political preferences through voting. Costs, on the other hand, include the possibility that contributors influence candidates' policy platforms or receive some kind of favors from them. Previous literature analyzing the welfare effects of campaign finance policies has focused exclusively on parties' expenditure of contributions on informative advertising. In this context, the benefit of advertising—and consequently of contributions—on citizens' welfare comes through the fact that it increases the probability of choosing better qualified candidates. Accordingly, the negative effect of campaign contributions on citizens' welfare is connected to policy favors or to ideological biases in the level of policy implemented (see Ashworth, 2006 for a detailed discussion of these models).

Assuming that campaign resources are spent on informative advertising is not the only plausible way of understanding their social benefits. A crucial use given to these resources, and consequently to contributions, is to mobilize citizens to vote. To do this, parties spend a significant amount of resources on activities aimed at internalizing (completely or partly) their sympathizers' voting costs (e.g., information, opportunity costs and transportation). Moreover, the advertisement of political platforms or candidate's characteristics can be understood as a way of reducing voters' information costs.

Taken this way, campaign contributions allow parties to provide valuable information about their policy platforms to citizens, which in turn reduces their cost of voting and encourages the participation of a larger and better informed electorate. Campaign contributions also reduce the other citizens' voting costs, allowing them—many of whom otherwise would not—to express their political preferences through voting. Consequently, campaign contributions have large social benefits: they allow for the possibility of having a more civically engaged and informed citizenry, which constitutes a valuable contribution to the electoral process and, ultimately, to society.

In this paper, we consider this alternative approach and analyze the welfare effect of a variety of campaign finance policies. Although previous studies have already highlighted the key role of campaign spending in voters' mobilization (see Herrera, Levine, and Martinelli, 2008), no previous work has analyzed how these contributions and campaign finance policies affect citizens' welfare along the lines here considered. As we show, doing so has significant consequences for the evaluation of the welfare of campaign finance policies.

We model parties' expenditure on mobilization through a "voting subsidy"—also referred here as the parties' mobilization technology—, and use expressive voting theory (Fiorina, 1976; Brennan and Hamlin, 1998; Schuessler, 2000; Hillman, 2010; Aldashev, 2015; among others) to account for the positive impact of this subsidy and, consequently, of campaign contributions on both voters mobilization and social welfare. In short, this theory argues that citizens obtain a direct utility from expressing their political preferences via voting. We use this *expressive utility* to measure the welfare gains from campaign contributions.

Additionally, we measure the social cost of contributions in terms of the opportunity cost of public funds. To do this, we consider that interest groups only contribute to parties' campaigns if they are rewarded contracts for public works in the event that the receiving party wins the election. We also refer to these interest groups as "contract-induced".[1] To model the opportunity cost of contributions, we assume that interest groups recover their campaign contributions by discounting these resources from the public resources they receive to execute public works. This subtraction of resources entails a welfare loss for citizens in terms of the quality of these works. More generally, by doing this, we aim to fit situations where a candidate and an interest group agree on a series of favors granted to the latter for their contributions, and these favors entail a direct opportunity cost for citizens in terms of the quality of public works. Despite some technical issues, this cost is conceptually similar to a standard "policy favor cost". For the same token, this cost is not necessarily related to corruption—although this could be the case—but to a welfare loss due to a favor exchange.

We introduce these elements into a political competition model. It considers two office-seeking parties of opposing ideologies; a continuum of citizens, each of which is assumed to be rational and expressive, and characterized as having a political ideology and a voting cost; and a large number of contractors, some of which (the interest groups) are contract-induced and have political attachments. At the start, each party bargains with respective contractors over the amount of campaign contributions they will make. These contributions determine the quality of public works to be provided by each party in the event that it wins the election. Contributions are then spent on voting subsidies. Observing this, and anticipating the quality of respective public works, citizens decide whether or not to vote and for whom to vote.

In our model, at equilibrium, unrestricted campaign contributions are above the efficient level if respective parties' mobilization technology is not sufficiently productive. This happens because, in this case, the relative valuation of the marginal benefit of contributions vis-a-vis their marginal cost is bigger for parties than for voters. We analyze the welfare effect of a variety of campaign finance policies when this happens. Because of the nature of our model, we not only consider the welfare of voters, but also that of abstainers. We refer to the members of both groups as citizens. In accordance with the previous literature, we find that banning contributions is only welfare-improving for citizens under certain parameters of the model—in our case, if the productivity of respective parties' mobilization technology is too low. We also find that limiting contributions is Pareto improving if this technology is not sufficiently productive.

More important, and contrary to previous results in the advertising literature, we obtain two novel results. First, we find that subsidizing campaigns with public funds and simultaneously banning contributions is welfare-improving for citizens and contractors (but not necessarily for contributors) only if the productivity of respective parties' mobilization technology is low enough. Furthermore, subsidizing campaigns with non-matching public grants—subsidies that do not depend on private contributions raised by parties—while simultaneously limiting contributions is Pareto improving under the same productivity circumstances. These results are driven by the fact that, in our framework, both public and private funds are equally useful for mobilizing voters, while non-matching public funds are

[1]A growing literature shows that firms specializing in public works can expect a substantial increase in the number of contracts they receive as they contribute to political campaigns (Boas, Hidalgo, and Richardson, 2014; Ruiz, 2017).

not useful for signaling respective candidates' quality in an advertising model. Nevertheless, subsidies are only useful if, as said above, the productivity of respective parties' mobilization technology is low enough.

Second, we find that simultaneously allowing for unrestricted contributions, imposing a contribution lump-sum tax on either parties or contractors, and investing these revenues on public works is always welfare-improving for citizens, but not for contractors. This occurs for two reasons. First, taxes positively affect the net contributions received by parties, which in turn positively affects their voting subsidies; second, investing these tax revenues in public-works projects more than compensates for the resources that contractors discount from their investments in them, consequently allowing for their high quality. Furthermore, combining this policy with a limit on contributions is always Pareto improving. These tax results hold regardless of the productivity level of parties' mobilization technology.

The closest papers to our work here are Prat (2002), Coate (2004a) and Ashworth (2006). These studies consider campaign contributions as being exclusively spent on informative advertising. The main difference between these studies is that the first considers advertising as containing non-direct information, while the latter two assume that advertising is directly informative.[2] This difference becomes important when evaluating the welfare effects of campaign finance policies. Publicly funding campaigns under non-direct information is useless because there is no signal if the government funds all candidates. Only under directly informative advertising are public matching grants (public subsidies that depend on the private contributions raised by each party) welfare-improving for voters. Nevertheless, all these studies share some similar results: either banning contributions or limiting contributions can be efficient policies depending on the specific parameters of each model. We find similar results for these policies in our context.

The differences between these studies and ours also become important when evaluating the welfare effect of campaign finance policies. As mentioned above, we find not only that under certain conditions, non-matching (rather than matching) public subsidies can achieve welfare improvements, but also identify that taxing contributions is useful for this purpose. To the best of our knowledge, taxing contributions has not been considered in previous welfare analyses similar to ours.[3]

More in general, our paper is relevant to previous literature that, like us, considers campaign contributions as a form of *quid pro quo* (Baron, 1994; Coate, 2004b; Gerber, 1996;

[2]Under non-directly informative advertising, voters are influenced by ads, not because of the messages they transmit but because of the amount of money spent on them (See Milgrom and Roberts, 1986). Under directly informative advertising, ads transmit verifiable information to voters that cannot be falsified (see Tirole and Jean, 1988 for a survey).

[3]Cotton (2009) analyzes the effect of taxing contributions on constituents' welfare under a different context; namely, one where politicians decide whether to sell policy favors or sell access to influence a unidimensional policy. This affects citizens' welfare, inasmuch as the contributions only benefit politicians and not citizens.

Grossman and Helpman, 1996; Potters, Sloof, and Van Winden, 1997; Prat, 2000; among others). Note that, unlike us, there is other literature that considers campaign contributions as a means of obtaining access to politicians—informational lobbying (Austen-Smith, 1987; Cotton, 2009; Schnakenberg, 2017; Bombardini and Trebbi, 2020; among others). We adopt the former approach and contribute to this literature by providing a novel model that connects it to the expressive voting theory via voters' mobilization. By doing so, we are also contributing to the literature on expressive voting cited above.

We organize the paper as follows: Section 2 describes the model; section 3 characterizes the unrestricted campaign contributions at equilibrium; section 4 analyzes the welfare properties of unrestricted contributions; and section 5 analyzes the welfare properties of a variety of campaign finance policies. The results are discussed in section 6, while section 7 concludes. The Appendix contains all the proofs.

2 Model.

2.1 Parties.

Consider two risk-neutral political parties competing for office, party L, the leftist party, and party R, the rightist party. Each aims to maximize its probability of victory. Ideology is measured on a 0 to 1 scale; the ideology of party L is 0, while that of party R is 1. Each party receives campaign contributions $C_k \in [0,1]$, with $k = \{L, R\}$, coming from a unique contract-induced contractor. We normalize to 1 the upper bound of C_k and return to this normalization below.

Parties invest campaign contributions on internalizing the voting cost of their sympathizers, and, consequently, in mobilizing them. We use the term "sympathizers" to refer to citizens that would be willing to vote for a party, but who do not necessarily vote in the elections because of voting costs. As is well known, voting entails costs to citizens, the most important of which are the cost of acquiring information, the opportunity cost of voting, and eventually even the transportation cost. To mitigate citizens' cost for acquiring information about candidates' characteristics and platforms, parties invest campaign contributions in publicizing this information through different channels (political meetings, advertising, mailings, email, door-to-door campaigns, etc.) By doing so, parties encourage the participation of a larger and better informed electorate. Similarly, parties might allocate resources to reduce the other voting costs, which would also allow citizens to be more involved in the social decision-making process. These are the benefits we aim to capture in our model.

We formalize parties' mobilization spending through a voting subsidy. We denote the voting subsidy that each party offers to each of their sympathizers in exchange for participating in the election (in terms of citizens' utility) as $\alpha(C_k)$, where $\alpha(.)$ is assumed to be a continuous, strictly concave increasing function of campaign contributions with $\alpha(0) = 0$, and $\alpha(1) = \bar{\alpha} \in (0, 1)$. As will be clear below, the assumption on the upper bound of $\alpha(.)$ implies that the subsidy technology does not allow parties to mobilize all citizens even if $C_k = 1$. Furthermore, we assume that $\lim_{C_k \to 0} \frac{\partial \alpha}{\partial C_k} = \infty$, and $\lim_{C_k \to 1} \frac{\partial \alpha}{\partial C_k} \in [0, \infty)$.

$\alpha(.)$ can also be understood as the parties' mobilization technology. This transforms campaign contributions into the same per capita subsidy for each citizen, and is assumed to exhibit decreasing returns to scale. Furthermore, $\bar{\alpha}$ can be understood as a measure of how productive the parties' mobilization technology is. Given the characteristics of $\alpha(.)$, the larger $\bar{\alpha}$ is, the greater the subsidy per unit of resources spent. As will become clear in section 2.3, this subsidy will allow voters to reduce their voting costs and, in the case of voting, obtain an *expressive utility.*

Remarkably, we assume that the voting subsidy offered by each party to its respective sympathizers is the same. In doing this, we implicitly assume that parties are unable to perfectly target each citizen with a personalized subsidy. This is compatible with the assumption that each citizen's characteristics are private information (see below).

Once in office, the winning party provides a unique public-works project of quality $Q \in [0, 1]$ that benefits citizens—where 1 is the maximum quality level. Said project can be understood as the provision of a public facility (the construction of roads, hospitals, schools or the delivery of health care, education, etc.). The party in office executes the project with the contractor who contributed to its campaign. The amount of public resources available to execute the project is normalized at 1. The upper bound of contributions follows from this normalization. We come back to the technology used to deliver the project in the next sub-section.

Parties' payoffs are given respectively by $B_L = \rho$, and $B_R = 1 - \rho$, where ρ stands for the probability of victory of L. As we will see, these probabilities are functions of campaign contributions.

2.2 Contractors.

There is a market of n risk-neutral contractors, where $n > 2$. From this set of contractors, only contractors 1 and 2 are contract-induced—i.e., they are willing to finance parties' campaigns if they are offered a contract to execute the public-work project in the event that their respective recipient party wins the election. In other words, these contractors see their contributions as an initial investment with uncertain returns. For political-ideological reasons, contractor 1 is only willing to contribute to party L, and contractor 2 is only willing to contribute to party R. The rest $n - 2$ contractors are never willing to contribute to any party—possibly also for ideological reasons.[4]

All contractors use the same technology to deliver the public works project. This technology is given by $Q(\delta, I) = \delta q(I)$, where $\delta \in (1, \bar{\delta})$, with $\bar{\delta} < \infty$, is the contractor's productivity, and $I \geq 0$ is the amount of resources the contractor invests in the project to execute it. We assume that $q(.)$ is a continuous strictly concave increasing function with $q(0) = 0$, $q(1) = 1$, $\lim_{I \to 0} \frac{\partial q}{\partial I} = \infty$, and $\lim_{I \to 1} \frac{\partial q}{\partial I} \in [0, \infty)$. Therefore, the quality of the project depends on the amount of resources the contractor invests in the project, which exhibits decreasing returns to scale. The contractor's profitability increases as its productivity (δ) increases.

The party in office does not have to compromise in terms of how it allocates the contract for the public-works project as it does not receive campaign contributions. The best we can assume in this case is that this party randomly chooses one of the n contractors (including

[4]One could consider that there are more than one contractor willing to contribute to each party. We discuss it below.

1 and 2) to execute the public-works project. The party in office pays this contractor 1, and demands a quality of the project equals 1. This implies that the contractor must invest $I^0 = q^{-1}(1/\delta) \in (0,1)$, and obtain profitability $\mu = 1 - I^0 \in (0,1)$. Consequently, citizens enjoy a project of quality 1 in this case.

Nevertheless, parties are willing to accept, in case of winning the election, a lower quality of the project in exchange for a contribution. Therefore, and before elections, each party k makes a take-it-or-leave-it offer to its respective contractor (1 or 2) in which (i) the contractor pays C_k to the party; (ii) the contractor is granted with the execution of the project if k wins; (iii) the contractor invests $I^0 - C_k$ in the project; (iv) the party accepts quality $\delta q(I^0 - C_k) < 1$ but still pays 1 to the contractor. If the contractor accepts this offer, her profits are still μ, since $1 - (I^0 - C_k + C_k) = 1 - I^0 = \mu$. The difference between $Q(\delta, I^0) = 1$ and $Q(\delta, I^0 - C_k) < 1$ represents the campaign contribution cost for citizens.

Therefore, the negotiation between parties and contractors takes place on the contributions space, taking as given the price of the project. Moreover, since the amount of the contribution determines the quality of the project, the negotiation is reduced to an agreement on C_k.

Each contractor decides to accept (A) or reject (N) her respective party's offer. If the contractor accepts the offer and her party wins the election, the agreement is implemented and the contractor obtains μ. Nevertheless, if her party loses, the contractor never receives a contract to execute the public procurement project and receives a payoff of $-C_k$. If the contractor rejects the offer, the party receives zero contributions. If this is the case and the party wins the election, it will randomly choose one of the $n-2$ never willing to contribute contractors to execute the project. Consequently, k's contractor receives a payoff of zero. Finally, a contractor receives a payoff of zero if she is never chosen to execute the project.

2.3 Citizens.

Each citizen is described by a pair of traits (γ, x). γ represents the citizen's political ideology, and is assumed to be uniformly distributed with support $[0,1]$. x represents the citizen's idiosyncratic cost of voting in terms of utility, and is assumed to be uniformly distributed with support $[0,1]$. Consequently, as anticipated above, $\bar{\alpha} < 1$ implies that it is not possible to mobilize all voters, even if $C_k = 1$. We assume that γ and x are independently distributed. The traits (γ, x) are private information, even though their distributions are common knowledge.

Before the election, citizens observe the subsidy $\alpha(C_k)$ offered by each party. Since citizens are rational, they can perfectly predict the quality of the public-works project. Thus, voters' expected value regarding the quality of the project is $Q(\delta, I^0 - C_k)$.

The utility of a citizen (γ, x) depends on two things: the voter's *policy utility*, and the *utility of voting*—which in turn depends on the *expressive utility*. Consider first the *policy utility*. Ignoring whether a citizen (γ, x) votes or not, her payoff if party L wins is given by:

$$U^L = -\gamma + \delta q(I^0 - C_L) \tag{1}$$

Accordingly, the payoff perceived by a citizen (γ, x) if party R wins is given by:

$$U^R = -(1-\gamma) + \delta q(I^0 - C_R) \tag{2}$$

For the sake of simplicity, Equations 1 and 2 assume that citizens equally value their political ideology and the quality of the public-works project.[5]

To model citizens' turnout and measure the social benefit of campaign contributions, we assume that citizens are expressive. As said above, expressive voting theory argues that citizens derive utility from expressing their political preferences through voting. We borrow from Aldashev (2015) his conception of how citizens perceive this *expressive utility*, and assume that this is given by $e\left|\frac{1}{2}-\gamma\right|$, where $e > 0$.[6] This utility implies that citizens care about their political ideology and derive utility from expressing that by voting. Furthermore, this implies that the *expressive utility* is higher the more ideologically minded the citizen. As Aldashev (2015) notes, the studies by Glaeser, Ponzetto, and Shapiro (2005) and Hortala-Vallve and Esteve-Volart (2011) give empirical support for this last assumption.[7]

The *expressive utility* is only perceived by citizens if they vote. Whether a citizen votes or not is explained by the comparison between her cost of voting and considerations pertinent to the expression of a particular preference in and of itself. Taking this into account and assuming that the accuracy of campaign targeting for each party is perfect, a citizen (γ, x) who votes for her preferred party, k, obtains a *voting utility* equal to:

$$e\left|\frac{1}{2}-\gamma\right| - x + \alpha(C_k) \tag{3}$$

Therefore, by internalizing all or part of their voting costs via the voting subsidy, campaign contributions allow citizens to express their ideological preferences through voting and

[5] A relative parameter for this valuation can be introduced in the model, without affecting the qualitative results.

[6] More precisely, we assume that $e \in (0, 4\bar{\alpha}]$. This guarantees that the amount of the campaign contribution determined between a party and a contractor is not zero. Only citizens with a high cost of voting and a moderate ideology are expected to abstain from voting if e is too high. Since mobilizing these citizens is too costly, parties will prefer zero contributions in this case. This is formally shown in the proof for Proposition 1 in the Appendix.

[7] Glaeser et al. (2005) find that only 45% of citizens who declare themselves as independent vote at the Presidential elections in USA, whereas 80% of citizens who strongly identify themselves as partisan do. Hortala-Vallve and Esteve-Volart (2011) find that the probability that a citizen with stronger positions on liberal-conservative scale vote is noticeably greater than the respective probability for their more ideologically neutral counterparts in USA.

thereby receive the associated *expressive utility*. We take this non-instrumental utility as a micro-founded way of capturing the marginal social benefit of campaign contributions which derives from having more and better-informed citizens involved in the electoral process.

The assumption of perfect accuracy of campaign targeting implies that each citizen only receives the subsidy from her preferred party if she votes. We come back to this assumption in Section 6.

A citizen who does not vote gets zero *voting utility*. The total payoff a citizen perceives is then given by the *policy utility* plus the *voting utility*.

2.4 Timing and definition of equilibrium.

The timing is as follows:

1. Each party simultaneously and independently makes the take-it-or-leave-it offer (C_k)—described in section 2.2—to its respective contractor (1 or 2).

2. Contractors 1 and 2 observe their respective offers and simultaneously and independently decide to accept (A) or reject (N) it.

3. Each party spends $C_k \geq 0$ on campaigning.

4. Observing the voting subsidies, each citizen simultaneously and independently decides which party to support and whether or not to vote.

5. Prior to the election, party L gets a random (valence) shock ϕ.

6. Elections are held and the party with the most votes wins the election. Each party wins with probability $\frac{1}{2}$ if both parties receive the same number of votes. If the winning party's contractor contributed to its campaign, she is contracted to execute the public project under the conditions agreed in stage 1; otherwise, the party randomly chooses one of the $n-2$ never willing to contribute contractors to execute the project.

If one allows for many contract-induced contractors willing to contribute to each party, the negotiation process is modified as follows: each party randomly chooses one of its contractors to finance the campaign and simultaneously and independently makes him/her a take-it-or-leave-it offer C_k. If the contractor accepts, the contribution is made, and the negotiation ends. If the contractor rejects the offer, the party randomly chooses another contract-induced contractor and the process is repeated. If the party cannot achieve an agreement with any of its contractors, it receives zero contributions and, in the event that it wins the election, it will randomly choose one of the never willing to contribute contractors to execute the project. This modification does not affect the final result of the bargaining process.[8]

[8] Determining whether interest groups have incentives to donate to just one campaign versus more than one is beyond the scope of this paper. These questions have been already addressed in the first wave of the literature on campaign contributions (See Morton and Cameron, 1992, for a survey on this literature.)

We define the sympathizer indicator $s_{(\gamma,x)} \in \{L, R\}$, where $s_{(\gamma,x)} = k$ if citizen (γ, x) prefers party k. We also define the participation indicator for each citizen as $\nu_{(\gamma,x)}$. It takes a value of 1 if citizen (γ, x) votes when contributions are (C_L, C_R), and 0 otherwise.

Definition 1 *An equilibrium in pure strategies is defined as: (1) a pair of campaign contributions (C_L^*, C_R^*), where C_k^* maximizes the party k's payoff subject to its respective contractor's participation constraint (on which more below); (2) a strategy in the space $\{A, N\}$ for each contractor 1 and 2 as a function of respective C_k; (3) an indicator $\nu^*_{(\gamma,x)}$ for each citizen as a function of respective C_k; and an indicator $s^*_{(\gamma,x)}$ for each citizen as a function of contributions (C_L, C_R).*

3 Equilibrium with unrestricted contributions.

We use backward induction to solve for the equilibrium.

3.1 Voting.

Consider the fourth stage. Citizen (γ, x) sympathizes with party L if $U^L \geq U^R$. Using Equations 1 and 2, it follows that this happens if and only if $\gamma \leq \frac{1+\Delta Q}{2}$, where $\Delta Q = \delta[q(I^0 - C_L) - q(I^0 - C_R)]$ is the difference in the quality of the public-works project to be provided by party L and party R.

The expected share of party L's sympathizers is then given by $Pr[\gamma \leq \frac{1+\Delta Q}{2}] = \frac{1+\Delta Q}{2}$. Correspondingly, the expected share of party R's sympathizers is given by $1 - \frac{1+\Delta Q}{2}$. We define:

$$\hat{\gamma}(C_L, C_R) = \frac{1 + \delta[q(I^0 - C_L) - q(I^0 - C_R)]}{2} \tag{4}$$

How $\hat{\gamma}$ changes as C_k changes measures the change in party L's sympathizers (and, consequently, in R's sympathizers) as the quality of the project provided by k changes.

Consider now the participation decision. Using Equation 3, it follows that citizen (γ, x) goes out and votes for her preferred party, k, if and only if $e\left|\frac{1}{2} - \gamma\right| - x + \alpha(C_k) \geq 0$. Using this and the distribution of x, we can compute the probability that citizen (γ, x) votes as:

$$P_{\gamma,x} = Pr\left[x \leq e\left|\frac{1}{2} - \gamma\right| + \alpha(C_k)\right] = e\left|\frac{1}{2} - \gamma\right| + \alpha(C_k) \tag{5}$$

with $P_{\gamma,x} = 1$ if $e\left|\frac{1}{2} - \gamma\right| + \alpha(C_k) > 1$.

The expected number of votes for party L (V_L) is given by the L's expected number of sympathizers, corrected for the probability of them actually turning out. This number can be computed as $V_L = \int_0^1$ *Pr(citizen (γ, x) sympathizes with L)*$P_{\gamma,x}\mathrm{d}\gamma = \int_0^{\hat{\gamma}} P_{\gamma,x}\mathrm{d}\gamma$. Similarly, the expected number of votes for party R can be computed as $V_R = \int_{\hat{\gamma}}^1 P_{\gamma,x}\mathrm{d}\gamma$. Using Equation 5 to compute these integrals, and assuming without a loss of generality that $\hat{\gamma} \geq \frac{1}{2}$—notice that this happens if and only if $C_L \leq C_R$—we obtain:

$$V_L(C_L, C_R) = \frac{e}{4} + \hat{\gamma}\left[\alpha(C_L) - (1 - \hat{\gamma})\frac{e}{2}\right] \tag{6}$$

$$V_R(C_L, C_R) = (1-\hat{\gamma})\left[\alpha(C_R) + \hat{\gamma}\frac{e}{2}\right] \tag{7}$$

Equations 6 and 7 define the expected number of voters for each party as a function of the vector of contributions (C_L, C_R). The first term in Equation 6 arises from the assumption that $\hat{\gamma} \geq \frac{1}{2}$.

The expected turnout is then given by $T = V_L + V_R$. Replacing Equation 6 and 7 in this expression, we get:

$$T(C_L, C_R) = \frac{e}{4} + \hat{\gamma}\alpha(C_L) + (1-\hat{\gamma})\,\alpha(C_R) \tag{8}$$

Each party k's expected vote share is computed as $\frac{V_k}{T}$.

The probability of victory of party L is given by the probability that its vote share is larger than $\frac{1}{2}$. At stage 5, prior to the election, the vote share of party L gets an additive valence shock ϕ, which is uniformly distributed on the interval $[-\frac{1}{2}, \frac{1}{2}]$. Correspondingly, the probability of victory of party L is given by:

$$\rho(C_L, C_R) = Pr\left[\frac{V_L}{T} + \phi \geq \frac{1}{2}\right] = \frac{V_L}{T} \tag{9}$$

Consequently, party R wins with probability $1-\rho(.)$. As anticipated before, these probabilities are functions of the vector of contributions.

There are two channels through which C_k affects the probability of victory of party k. First, through the effect of C_k on the voting subsidy (*mobilization channel*). Although an increase in C_k positively affects both V_k and T through this channel, this *mobilization channel* is always positive.[9] Second, through the effect of C_k on the quality of the public-works project, and consequently on the proportion of sympathizers (*sympathizer channel*). On the one hand, increasing C_k negatively affects V_k, and consequently negatively affects party k's probability of victory. On the other hand, the effect of an increase in C_k on T depends on whether a party is receiving more or fewer contributions than the other—this comes from the assumption that $\hat{\gamma} \geq \frac{1}{2}$. This effect is negative for party L and ambiguous for party R.[10] However, as we will see later, the effect of an increase in C_k on T through this channel and for both parties becomes irrelevant in equilibrium as this is symmetric. Consequently,

[9]Using the log of the probability of victory of each party, this effect for party L is given by $\frac{\partial ln\rho}{\partial\alpha(C_L)}\frac{\partial\alpha(C_L)}{\partial C_L} = \frac{\hat{\gamma}}{V_L}(1-\rho)\frac{\partial\alpha(C_L)}{\partial C_L} > 0$. This effect for party R is given by $\frac{1-\hat{\gamma}}{V_R}\rho\frac{\partial\alpha(C_R)}{\partial C_R} > 0$.

[10]Using the log of the probability of victory of each party, this effect for party L is given by $\frac{\partial ln\rho}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial C_L} = \frac{1}{V_L}\left[\alpha(C_L) + e(\hat{\gamma}-\frac{1}{2})\right]\frac{\partial\hat{\gamma}}{\partial C_L} - \frac{1}{T}[\alpha(C_L) - \alpha(C_R)]\frac{\partial\hat{\gamma}}{\partial C_L} < 0$. This effect for party R is given by $-\frac{1}{V_R}\left[\alpha(C_R) + e(\hat{\gamma}-\frac{1}{2})\right]\frac{\partial\hat{\gamma}}{\partial C_R} - \frac{1}{T}[\alpha(C_L) - \alpha(C_R)]\frac{\partial\hat{\gamma}}{\partial C_R}$. The sign of the first term is negative and the sign of the second one is non-negative; so the final effect is ambiguous. In a symmetric equilibrium, this second term and the second one in the effect of party L become zero.

in an equilibrium, contributions will always negatively affect party k's probability of victory through this *sympathizer channel*.

3.2 Campaign Contributions.

Consider the second stage. On the one hand, if 1 and 2 accept their respective party's offer, they receive payoffs $\pi_1 = \rho\mu - (1-\rho)C_L$, and $\pi_2 = (1-\rho)\mu - \rho C_R$. On the other hand, since 1 and 2 are never contracted to execute the project if they reject their respective party's offer, each of them receives a payoff equal to zero as this occurs. Consequently, contractor 1 accepts party L's offer if and only if:

$$\frac{C_L}{\mu + C_L} \leq \rho \tag{10}$$

Similarly, contractor 2 accepts party R's offer if and only if:

$$\frac{C_R}{\mu + C_R} \leq 1 - \rho \tag{11}$$

Consider now the first stage. Party L chooses its campaign contribution offer by solving the following program:

$$\underset{C_L \in [0,1]}{\text{Max}} \ \rho \tag{12}$$

$$\text{s.t. equations } (4), (6), (8), (9), (10).$$

Similarly, party R chooses its campaign contribution offer by solving:

$$\underset{C_R \in [0,1]}{\text{Max}} \ (1-\rho) \tag{13}$$

$$\text{s.t. equations } (4), (7), (8), (9), (11).$$

Participation constraints (Equations 10 and 11) put a lower bound on each party maximand. We refer to the contributions that solve 12 and 13 (C_L^*, C_R^*) as the **unrestricted campaign contributions**.

3.3 Equilibrium.

Proposition 1
Under unrestricted campaign contributions there is a unique equilibrium (in pure strategies) in which both parties demand the same contributions, and the probability of victory for each party is $\frac{1}{2}$. If the contractors participation constraints—Equations 10 and 11—are not binding, then $C_L^* = C_R^* = C^*$, where $C^* \in (0, \mu)$ is implicitly defined as:

$$\delta \frac{\partial q(I^0 - C^*)}{\partial I} = \frac{1}{2\alpha(C^*)} \frac{\partial \alpha(C^*)}{\partial C} \tag{14}$$

Otherwise, $C_L^* = C_R^* = \mu$.

At this unique equilibrium, each contractor accepts his party's offer; $\nu_{(\gamma,x)} = 1$ if the expression in Equation 3 evaluated at (C_L^*, C_R^*) is larger or equal to zero, and $\nu_{(\gamma,x)} = 0$ otherwise; and $\hat{\gamma}^*(C_L^*, C_R^*) = \frac{1}{2}$, which means that half the citizens sympathize with L, and the remaining half sympathize with R. Moreover, the quality of the project is given by $q(I^0 - C^*)$, while the voting subsidy is given by $\alpha(C^*)$.

We say an equilibrium is interior if $C_k^* \in (0, \mu)$ for each k. As it will become clear below, corner solutions arise when the productivity of contractors is very low. For our analysis, we assume that this is not the case and concentrate on the unique interior equilibrium. Imposing this is similar to assuming that our results are not driven by low contractors' productivity.

The left hand-side of Equation 14 multiplied by $\alpha(C^*)$ represents party k's marginal cost of campaign contributions—notice that we are leaving the term $\alpha(C^*)$ to the right hand-side of this equation. This marginal cost comes from the *sympathizer channel*, and in particular from the party k's loss in its expected number of voters (V_k) due to an increase in contributions.[11] The right-hand side term of Equation 14 multiplied by $\alpha(C^*)$ represents party k's marginal benefit of campaign contributions. This comes from the *mobilization channel*. Equation 14 simply says that at equilibrium, the optimal contributions are such that each party's marginal cost of contributions is equal its marginal benefit.

Figure 1 depicts the marginal cost and marginal benefit of campaign contributions for party k (both multiplied by $\frac{1}{\alpha(.)}$) in an interior equilibrium. Party k's marginal benefit decreases as C_k increases, and party k's marginal cost increases as C_k increases (the proof for Proposition 1 in the Appendix formally shows the properties of these functions.) The unique interior equilibrium occurs where the two curves intersect.

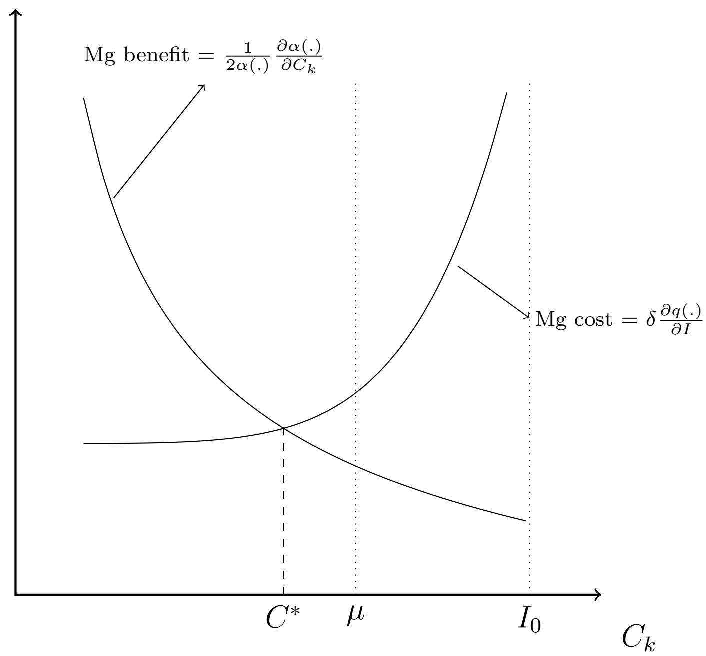
Lemma 1
In an interior equilibrium, unrestricted campaign contributions (C^*) decrease as contractors productivity (δ) increase.

The result in Lemma 1 follows immediately from the inspection of Figure 1. This result also shows that corner solutions emerge when the productivity of contractors is very low.

Figure 2 depicts citizens' behavior—in the space (x, γ)—in both the presence and absence of campaign contributions. The solid function divides citizens between those who vote (those in regions A, B, C and D) and those who do not vote (those in region E) when campaign contributions are present. The dashed function separates citizens between those who vote (those in regions A and C) and those who do not vote (those in regions B, D, and E) when campaign contributions are absent. Thus, campaign contributions positively affect turnout. These not only mobilize citizens with high ideological attachments and high voting costs, but also those with low ideological attachments and low voting costs. This comes, however, at the cost of a decrease in the quality of public-works projects. These functions also separate those citizens who vote for party L (those in regions A and B when contributions are present; and in region A when they are absent) from those who vote for party R (those in regions C

[11] As anticipated above, since the equilibrium is symmetric, the effect of campaign contributions on T through the *sympathizer channel* becomes zero.

Figure 1: Interior Equilibrium



and D when contributions are present; and in region C when they are absent). The slope of the functions depicted in Figure 2 is $-e$ for $\gamma < \frac{1}{2}$, and e for $\gamma > \frac{1}{2}$. Thus, expressiveness does matter in determining citizens' participation at an interior equilibrium: turnout increases as e increases.

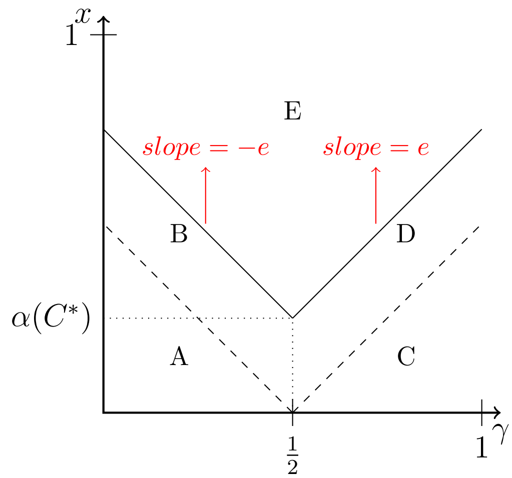
4 Welfare Properties of Unrestricted Contributions.

There are four groups of agents in our framework: parties, contractors, citizens who vote, and citizens who do not vote (abstainers). To analyze how the welfare of citizens changes as campaign contributions move away from C^*, it is useful to classify them as follows. Always-abstainers: citizens who do not vote at contributions C^*, and who still abstain from voting as contributions move away. Always-voters: citizens who vote at contributions C^*, and who still vote as contributions moved away. Switchers: citizens who vote at contributions C^*, but do not vote as contributions move below C^*; or who do not vote at contributions C^*, but do vote as contributions move above C^*.

At equilibrium, the party's expected payoff is given by $B_k^* = \frac{1}{2}$; the contractor's expected payoff is $\pi_g^* = \frac{1}{2}(\mu - C^*)$ for $g = 1, 2$, and $\pi_g = 0$ for $g = 3, ..., n$; and the expected payoff for the citizens (γ, x) who vote is $w_v^* = -\frac{1}{2} + \delta q(I^0 - C^*) + e\left|\frac{1}{2} - \gamma\right| - x + \alpha(C^*)$, and for the citizens (γ, x) who do not vote is $w_{nv}^* = -\frac{1}{2} + \delta q(I^0 - C^*)$.

The marginal benefit and the marginal cost of campaign contributions are not the same for parties and citizens. The marginal benefit of campaign contributions for always-voters is just given by the change in the voting subsidy as contributions increases. Furthermore, this marginal benefit is zero for citizens who do not vote. The marginal cost of campaign contributions for all citizens is just given by the change in the quality of the project. Even

Figure 2: Voting behavior and Campaign Contributions



more important, the relative valuation of the marginal benefit of campaign contributions vis-a-vis its marginal cost is not necessarily the same for parties and voters. The term $\frac{1}{2\alpha(C^*)}$ to the right of Equation 14 measures the difference in this valuation. This valuation is smaller for voters than for parties if this term is greater than one, and vice versa.

It is easy to anticipate how the payoffs of some groups of agents change as campaign contributions move away from C^*. First, parties will always obtain a payoff equal to 1/2 if they receive the same amount of contributions, regardless of whether these are above or below C^*. Second, since the project's profitability remains constant as contributions change, the expected payoffs for contractors 1 and 2 increase as contributions are reduced below C^*. The opposite happens as contributions increase above this level, and contractors are still able to contribute. Third, the payoffs for contractors 3 to n are always zero for any positive level of contributions. Finally, since the payoff for always-abstainers only depends (positively) on the quality of the project, reducing contributions below C^* always positively affects their payoffs, and vice-versa.

Therefore, increasing contributions above C^* never Pareto dominates unrestricted campaign contributions. At least always-abstainers and contractors 1 and 2 are worse off as this occurs. Nevertheless, as we formally proved in the Appendix (see the proof for Proposition 2), a small reduction in contributions below C^* positively affects the payoffs for always-voters and switchers if and only if, at equilibrium, $\frac{1}{2\alpha(C^*)} > 1$; i.e., if and only if the relative valuation of the marginal benefit of campaign contributions vis-a-vis its marginal cost is lower for voters than for parties. Proposition 2 formalizes these results.

Proposition 2
a) Unrestricted campaign contributions are not Pareto efficient if $\alpha(C^*) < \frac{1}{2}$. A small and similar reduction in each party's campaign contributions below C^* Pareto dominates unrestricted contributions.
b) Unrestricted campaign contributions are not Pareto dominated for any other contribution amount $C \in [0, 1]$ if $\alpha(C^*) \geq \frac{1}{2}$.

Whether unrestricted contributions are Pareto efficient or not depends on the parties' mobilization technology. If this technology is not highly productive, namely if $\bar{\alpha} < \frac{1}{2}$, then $\alpha(C^*)$ is always smaller than $1/2$, regardless of the value of C^*, and unrestricted contributions are also always inefficient. Nevertheless, if this technology is highly productive, namely if $\bar{\alpha} > \frac{1}{2}$, then it could be possible to observe $\alpha(C^*) > 1/2$. For obvious reasons, we concentrate our welfare analysis of campaign finance policies on the case where unrestricted campaign contributions at equilibrium are inefficient. More precisely, we impose $\bar{\alpha} < \frac{1}{2}$.

5 Campaign finance policies

Banning contributions, limiting contributions, funding campaigns with public resources, or a combination of some of these are the most common campaign finance policies adopted around the world (see Falguera, Jones, and Ohman, 2014 for a description of the adoption of these policies across countries). In this section, we report our results regarding the welfare implications of these policies. Furthermore, we also report the welfare properties of taxing campaign contributions, which is not a common practice around the world.

We mainly concentrate our analysis on whether moving from unrestricted contributions to the policy under consideration is *welfare-improving* for each group. We say that a campaign finance policy is *welfare-improving* for a group of agents—vis-a-vis unrestricted contributions—if implementing this policy either positively affects the payoffs of all agents in the group or positively affects the payoffs of some of them without making at least one agent in the group worse off. Furthermore, we also analyze complementarities between policies.

We restrict our analysis to the case where campaign finance policies are such that both parties always receive the same amount of contributions. For this reason, we do not worry about them in the analysis of the results. Furthermore, since contractors' payoffs essentially depend on whether they are chosen to execute the public-works project or not, and always-abstainers' payoffs just depend on project quality, we will only care about these groups when needed.

5.1 Banning and limiting

Proposition 3

Suppose that $\bar{\alpha} < \frac{1}{2}$. Then:

a) Banning contributions is unambiguously welfare-improving for citizens and contractors, except for 1 *and* 2*, if $\bar{\alpha} < 1 - \delta q(I^0 - C^*) \in (0,1)$. It is not welfare-improving for always-voters and switchers if $\alpha(C^*) > 1 - \delta q(I^0 - C^*)$.*

b) Imposing a campaign contribution limit $\bar{C} \in [C^E, C^*)$, where $C^E \in (0, C^*)$ is such that $\frac{\partial \alpha(C^E)}{\partial C} = \delta \frac{\partial q(I^0 - C^E)}{\partial I}$, Pareto dominates unrestricted contributions. Furthermore, imposing $\bar{C} = C^E$ Pareto dominates any other limit in the interval $[C^E, C^*)$.

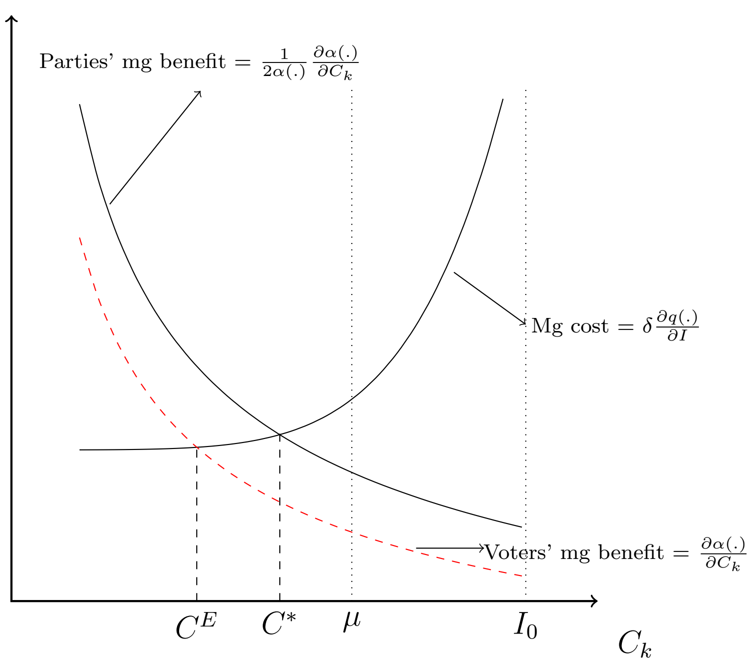
As explained before, the winning party does not have to compromise in terms of how it allocates the contract for the public-works project, as contributions are banned. Consequently, the winning party randomly chooses one of the n contractors (including 1 and 2) to execute the public-works project, and contracts a project of quality 1 with the chosen contractor. Therefore, $\delta q(I^0) = 1$, and the voting subsidy is then given by $\alpha(0) = 0$.

First, banning contributions is *welfare-improving* for contractors 3 to n—who obtain now a positive expected payoff. Nevertheless, this has an ambiguous effect on the payoffs for contractors 1 and 2. Second, banning contribution is *welfare-improving* for always-voters and switchers if and only if the increase in the quality of the project is greater than the reduction in the voting subsidy as contributions are banned, otherwise it is not. The inequalities in Proposition 3(a) follow from this reasoning (see the proof for Proposition 3(a) in the Appendix). Notably, banning contribution is *welfare-improving* for these citizens if the productivity of the parties' mobilization technology ($\bar{\alpha}$) is low enough. In other words, there always exists a low enough productivity of mobilization such that banning contributions is always *welfare-improving* for these groups.

Consider now the result in Proposition 3(b). When a contribution limit $\bar{C} \in (0, C^*)$ is imposed, contractors 1 and 2 are still willing to contribute the cap to their respective parties. Since they are contributing, they are still rewarded with the contract if their respective party wins office. The quality of the public-works project and the voting subsidy are then given by $\delta q(I^0 - \bar{C}) < 1$ and $\alpha(\bar{C})$, respectively.

Figure 3 illustrates the result in Proposition 3(b)—and, by the way, the result in Proposition 2(a). This is similar to Figure 1 but includes the marginal benefit of campaign contribution for voters. Our assumption on $\bar{\alpha}$ implies that $\frac{1}{2\alpha(C^*)} > 1$; i.e., that the relative valuation of the marginal benefit of contributions vis-a-vis the respective marginal cost is lower for voters than for parties. Hence, any decrease in contributions below C^* and up to C^E is *welfare-improving* for citizens. Furthermore, the optimal level of contributions for always-voters and switchers is C^E. Figure 3 also shows that even if banning contributions is *welfare-improving* for citizens—relative to C^*, this policy is an overcorrection of unrestricted campaign contributions.

Figure 3: Limiting Contributions—$\bar{\alpha} < \frac{1}{2}$



5.2 Public funding

To analyze the welfare effect of funding campaigns with public resources, we consider the following exercise. We take an amount $\beta \in (0, 1)$ from the resources available for contracting the public project and divide it equitably between the two parties in order to subsidize their campaigns. Hence, each party receives public resources $\beta/2$ to fund their campaigns, while the available resources to invest in the public-works project are reduced from 1 to $1 - \beta$. We begin by comparing this policy with unrestricted campaign contributions. Subsequently, and taking advantage of the results in Proposition 3, we analyze whether subsidizing campaigns can be a welfare complement to banning or limiting contributions. Proposition 4 presents the results of the former analysis.

Proposition 4
Suppose that $\bar{\alpha} < \frac{1}{2}$. *Then:*

a) Subsidizing parties' campaigns with public funds β *while simultaneously banning contributions is unambiguously welfare-improving for citizens and contractors, except for* 1 *and* 2, *if there exist at least one* $\beta < C^*$, *such that* $\bar{\alpha} < \delta[q(I^0 - \beta) - q(I^0 - C^*)] + \alpha(\beta/2)$. *This policy always exits if, in addition to satisfying this condition, the* β *that maximizes the welfare of always-voters lies in the interval* $(0, C^*)$. *This policy is never welfare-improving for citizens except for always-abstainers if* $\alpha(C^*) > \delta[q(I^0 - \beta) - q(I^0 - C^*)] + \alpha(\beta/2)$ *for all* $\beta \in (0, C^*)$.

b) Consider subsidizing parties' campaigns with public funds $\tilde{\beta}$ *while simultaneously imposing a campaign contribution limit* $\tilde{C} \in (0, C^*)$. *This policy is Pareto improving if there exist at least a pair* $(\tilde{C}, \tilde{\beta})$, *with* $\tilde{C} + \tilde{\beta} < C^*$, *such that* $\bar{\alpha} < \delta[q(I^0 - \tilde{\beta} - \tilde{C}) - q(I^0 - C^*)] + \alpha(\tilde{C} + \tilde{\beta}/2)$. *An infinite number of Pareto improving policies* $(\tilde{C}, \tilde{\beta})$ *exists if policy* β *exists in Proposition 4(a). Policy* $(\tilde{C}, \tilde{\beta})$ *is never welfare-improving for citizens except for always-abstainers if* $\alpha(C^*) > \delta[q(I^0 - \tilde{\beta} - \tilde{C}) - q(I^0 - C^*)] + \alpha(\tilde{C} + \tilde{\beta}/2)$ *for all* $(\tilde{C}, \tilde{\beta})$, *with* $\tilde{C} + \tilde{\beta} < C^*$.

Proposition 4(a) analyzes the welfare effect of subsidizing each party's campaign while simultaneously banning contributions. Since contributions are banned and the public resources available to contract the project is $1 - \beta$, the winning party awards a contract for a project of quality $\delta q(I^0 - \beta)$ with a randomly chosen contractor. Furthermore, the voting subsidy is $\alpha(\beta/2)$.

This finance policy is *welfare-improving* for always-abstainers if and only if $\beta < C^*$. As we show in the Appendix (see the proof for Proposition 4(a)), this policy is also *welfare-improving* for always-voters and switchers if and only if the increase in the quality of the project is greater than the reduction in the voting subsidy as the policy is implemented—i.e., $\alpha(C^*) < \delta[q(I^0 - \beta) - q(I^0 - C^*)] + \alpha(\beta/2)$. Otherwise, it is not. Proposition 4(a) follows from this reasoning. Once again, subsidizing campaigns while simultaneously banning contributions is *welfare-improving* for citizens if the parties' mobilization productivity ($\bar{\alpha}$) is low enough.

Proposition 4(b) analyzes the welfare effect of subsidizing each party's campaign with public funds while simultaneously limiting private contributions. Since the limit is below the unrestricted contribution level, contractors 1 and 2 contribute just the cap to their respective party. Consequently, the quality of the public-works project is given by $\delta q(I^0 - \tilde{C} - \tilde{\beta})$. The voting subsidy is $\alpha(\tilde{C} + \tilde{\beta}/2)$.

This finance policy is *welfare-improving* for always-abstainers if and only if $\tilde{C} + \beta < C^*$. As we show in the Appendix (see the proof for Proposition 4(b)), this policy is *welfare-improving* for always-voters and switchers if and only if the increase in the quality of the project is greater than the reduction in the voting subsidy as the policy is implemented—i.e., if and only if $\alpha(C^*) < \delta[q(I^0 - \beta - \tilde{C}) - q(I^0 - C^*)] + \alpha(\tilde{C} + \beta/2)$. Otherwise, it is not. Proposition 4(b) follows from this reasoning. Notably, publicly funding while simultaneously limiting campaign contributions is also *welfare-improving* for citizens only if the parties' mobilization productivity ($\bar{\alpha}$) is low enough.

We now consider the "complementarity" between the policies considered in section 5.1 and subsidizing campaign contributions. Suppose first that banning contributions is *welfare-improving* for citizens, and that they are indeed banned. Let us now introduce a subsidy β. As this happens, the quality of the public-works project decreases, negatively affecting the payoff of always-abstainers. Consequently, it does not improve the welfare of all citizens. Nevertheless, the subsidy would improve the welfare of always-voters and switchers. In this case, the optimal β can be obtained by equating the voter' marginal benefit of campaign contributions with their respective marginal cost.[12]

Suppose now that there is a limit C^E on campaign contributions. We already know from Proposition 3(b) that this limit is *welfare-improving* for citizens. Let us introduce a subsidy β. Since the marginal benefit and the marginal cost of campaign contributions for voters are equalized at C^E, introducing the subsidy causes their benefits to increase less than their costs. Consequently, it does not improve the welfare of these groups.

5.3 Taxing

We analyze now the welfare properties of taxing contributions and investing these resources in public-works projects. We center our analysis on lump-sum taxes; it can be verified that our results are similar if one allows for a contribution linear tax rate. Furthermore, we focus on tax rates that leave the unrestricted contributions with taxes in an interior equilibrium or just on its upper bound. We come back on this point below.

Taking money from contributors and investing these resources in public-works projects positively affects citizens' welfare. Nevertheless, the final effect of taxing contributions on the welfare of all groups of agents depends on how unrestricted contributions with taxes compare with unrestricted contributions without taxes at equilibrium. Proposition 5 solves these issues.

Proposition 5
Assume that $\bar{\alpha} < \frac{1}{2}$.

a) Allowing for unrestricted contributions, while also imposing a contribution lump-sum tax $\tau \in (0, \bar{\tau})$ *on either parties or their contractors, and then investing the resulting tax revenues on the public-works project, is welfare-improving for citizens.* $\bar{\tau}$ *is the maximum tax*

[12]This proof is available upon request.

rate, whereby, in each case (whether taxing the parties or their contractors), the unrestricted contributions with taxes are interior at equilibrium.

b) Consider simultaneously imposing a campaign contribution limit $\hat{C} \in (0, C^*)$; a contribution lump-sum tax $\hat{\tau} \in (0, \bar{\tau})$ on either the parties or their contractors; and then investing the resulting tax revenues on the public-works project. There exist an infinite number of policies $(\hat{C}, \hat{\tau})$ that are Pareto improving. For instance, policies satisfying $\hat{C} - \hat{\tau} = C^E$, if the tax is imposed on the parties; or $\hat{C} = C^E$, if the tax is imposed on contractors, are Pareto improving.

Let us begin by analyzing the case where unrestricted contributions are allowed and a lump-sum tax on either parties or their contractors is imposed. On the one hand, the voting subsidy is given by $\alpha(C_k - \tau)$, and the quality of the public-works project is given by $\delta q(I^0 - C_k + 2\tau)$ as the tax is imposed on parties. On the other hand, the voting subsidy is given by $\alpha(C_k)$, and the quality of the public-works project is given by $\delta q(I^0 - C_k + \tau)$ as the tax is imposed on contractors. Regardless of what the case is, solving for the unrestricted contributions with taxes, we find that, at equilibrium, each party demands and receives the same amount of contributions (see Equation 5A and 6A in the Appendix). We call these equilibrium contributions C^τ, which are a function of τ. As we show in the Appendix (see the proof for Proposition 5(a)), both the amount of the subsidy and the quality of the public-works project are the same (given the same τ), regardless of on whom the tax is levied.

The result in Proposition 5(a) follows from the following reasoning. Anticipating the tax and the possibility of compensating voters for the project-quality cost generated by contributions, parties demand higher net contributions to contractors ($C^\tau - \tau > C^*$). This, in turn, allows parties not only to offer a bigger voting subsidy but also a bigger quality of the public-works project to citizens. Nevertheless, this reasoning is valid only if the winning party is allowed to redistribute resources from the losing party's contractor to citizens. As it can be verified, imposing this tax on contributions, while simultaneously investing only up to τ (half of the tax revenues raised or less) on the public-works project, is never *welfare-improving* for citizens. When this happens, parties adjust their demand for contributions in such a way that, although they still demand more contributions from contractors (i.e., $C^\tau > C^*$), they are unable to obtain larger net contributions (i.e., $C^\tau - \tau \leq C^*$). The result is that parties offer neither a higher voting subsidy nor a better quality of the public-works project to citizens.[13]

It can be also verified that C^τ increases as τ increases. Therefore, $\bar{\tau}$ is implicitly given by the tax rate that leaves contributions just in their upper limit of an interior solution (μ if the tax is imposed on the parties; $\mu - \tau$ if it is imposed on contractors 1 and 2). Consequently, $\bar{\tau}$ maximizes the payoff for all groups of citizens subject to having an interior solution for C^τ. Nevertheless, this also happens to be the tax rate that most negatively affects the payoffs of contractors 1 and 2. Increasing the lump-sum tax rate beyond $\bar{\tau}$ leaves contributions unchanged at their upper limit, increases the quality of the public-works project, but decreases the voting subsidy. Eventually, it will negatively affect the welfare of always-voters and switchers.

[13] These proofs are available upon request.

Proposition 5(b) states that taxing contributions while simultaneously limiting contributions is Pareto improving. Furthermore, it introduces some (infinite) possible designs for this policy. Notably, as we show in the Appendix (see the proof for Proposition 5(b)), these designs guarantee that the welfare level achieved for all groups of citizens under these policies is greater than the one achieved when only imposing a limit C^E on contributions.

Whether taxing contributions without limiting them is better or worse for citizens than taxing and simultaneously limiting them depends on the specific form of both the mobilization technology and the quality technology. Comparing with the latter policy, the former offers a bigger voting subsidy but a smaller quality of the public-work project. This occurs because $C^{\tau} - \tau > C^E = \bar{C} - \hat{\tau}$.

6 Discussion

The aim of this section is twofold. First, we come back to our assumption of perfect accuracy of campaign targeting, and briefly discuss the effect of relaxing that. Second, we discuss some welfare optimality issues regarding the finance policies we analyzed.

So far, we have explicitly assumed perfect accuracy of campaign targeting, or, in other words, that parties can perfectly target their sympathizers with the voting subsidy. A way of relaxing this assumption is to assume that party k's sympathizers receive the voting subsidy from their preferred party with an exogenous probability smaller than one, and receive it from the opposite party with the complement probability. Correspondingly, the accuracy of campaign targeting decreases as the former probability decreases. Understood in this way, imperfect accuracy produces a positive campaign spending externality across parties that becomes greater as said accuracy decreases.

It can be verified that under imperfect accuracy of campaign targeting, parties still demand positive campaign contributions from their contractors if the accuracy level is not too low, or, in other words, if the externality is not too high. Due to this externality, parties will demand fewer campaign contributions from their respective contractors vis-a-vis the amount they would demand under perfect accuracy. Furthermore, there exists a level of externality where contributions can be efficient even if the mobilization technology is not highly productive. More importantly for our purpose, it can be verified that our welfare results regarding campaign finance policies still hold under imperfect accuracy, subject to observing positive contributions and having an externality level for which contributions are inefficient at equilibrium.[14]

Let us now briefly discuss the social optimality of the analyzed finance policies. So far, we have only concentrated on their efficiency properties. First, it does matter whether we focus our analysis on policies that are Pareto improving or just *welfare-improving* for citizens. Second, regardless of the chosen space, this social optimality analysis is not simple, because of the existence of many groups with different objective functions. For instance, imposing a tax on contributions could maximize the welfare of always-voters and switchers, but not that of always-abstainers, whose preferred level of contributions is zero. Nevertheless, we can make some comparisons of the welfare level of each group across different policies.

[14] These results are available upon request.

Consider the set of policies that are Pareto improving. There are at least two arguments that suggest that taxing and simultaneously limiting contributions might be desirable from an optimal point-of-view. First, as we already know, limiting contributions and subsidizing campaigns with public funds is only useful if parties' mobilization technology is not especially productive. Even assuming this to be the case, and accepting therefore that publicly funding campaigns is useful, analyzing which policy provides, say, more aggregate welfare still depends on the precise functional form of both the mobilization technology and the quality technology. Contrary to that, both limiting contributions, and taxing while simultaneously limiting contributions, are Pareto improving regardless of the parties' mobilization technology level. Second, as we have already mentioned in our analysis of Proposition 5, taxing and simultaneously limiting contributions provides more welfare to voters than just limiting contributions. These two arguments also apply to policies that are *welfare-improving* for all groups except for contributors.

7 Conclusions.

Unlike the previous literature in this field, we have analyzed the role of campaign finance policies in a context where parties spend private contributions on mobilizing votes rather than on informative advertising. Additionally, we concentrate on a particular type of policy favor, namely the allocation of public-works contracts, which allows us to measure the cost of these favors in terms of the opportunity cost of public resources.

We show that considering this alternative approach has significant consequences on the welfare evaluation of campaign finance policies. Comparing our results to previous findings, we not only find that non-matching public subsidies might be useful for achieving efficiency when the parties' mobilization technology is not especially productive, but also identify that taxing contributions is always useful for achieving this regardless of the productivity of mobilization. Combining each of these policies with limits on contributions is Pareto efficient.

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Appendix

Proof for Proposition 1.

We already know from the discussion in the main text that: (1) $s^*_{(\gamma,x)} = L$ if $\gamma \leq \hat{\gamma}$, and $s^*_{(\gamma,x)} = R$ otherwise; (2) $\nu^*_{(\gamma,x)} = 0$ if $e\left|\frac{1}{2} - \gamma\right| < x - \alpha(C_k)$, and $\nu^*_{(\gamma,x)} = 1$ otherwise; and (3) contractor 1 chooses A if and only if 10 holds, and N otherwise, and contractor 2 chooses A if and only if 11, and N otherwise.

Let us now consider parties' contribution offers. We begin by considering interior solutions. Solving the programs in 12 and 13, and assuming that the respective constraints are not binding, the parties' optimal campaign contribution offers are implicitly defined by $\frac{\partial V_L(.)}{\partial C_L} = \frac{\partial T(.)}{\partial C_L}\rho$, and $\frac{\partial V_R(.)}{\partial C_R} = \frac{\partial T(.)}{\partial C_R}(1-\rho)$. We show first that at an interior equilibrium, $C_L = C_R$. Using Equations 6, 7 and 8 to compute $\frac{\partial V_L}{\partial C_L}$, $\frac{\partial V_R}{\partial C_R}$, $\frac{\partial T}{\partial C_L}$, and $\frac{\partial T}{\partial C_R}$, by plugging these derivatives into the equilibrium conditions, and taking into account that $-\frac{\partial \hat{\gamma}}{\partial C_L} = \frac{\partial \hat{\gamma}}{\partial C_R} = \frac{\delta}{2}\frac{\partial q}{\partial I}$, the equilibrium conditions can be written as:

$$\frac{\delta}{2}\frac{\partial q(I^0 - C_L^*)}{\partial I}\left[\alpha(C_L^*)(1-\rho) + \alpha(C_R^*)\rho + e\left(\hat{\gamma} - \frac{1}{2}\right)\right] = \frac{\partial \alpha(C_L^*)}{\partial C_L^*}\hat{\gamma}(1-\rho) \tag{1A}$$

$$\frac{\delta}{2}\frac{\partial q(I^0 - C_R^*)}{\partial I}\left[\alpha(C_L^*)(1-\rho) + \alpha(C_R^*)\rho + e\left(\hat{\gamma} - \frac{1}{2}\right)\right] = \frac{\partial \alpha(C_R^*)}{\partial C_R^*}(1-\hat{\gamma})\rho \tag{2A}$$

Combining 1A and 2A, it follows that at an interior equilibrium:

$$\frac{\partial \alpha(C_L^*)/\partial C_L^*}{\partial \alpha(C_R^*)/\partial C_R^*}\hat{\gamma}(1-\rho) = \frac{\partial q(I^0 - C_L^*)/\partial I}{\partial q(I^0 - C_R^*)/\partial I}(1-\hat{\gamma})\rho \tag{3A}$$

Assume that at equilibrium, $C_L < C_R$. If this happens, the following three facts are true: First, $\hat{\gamma} > \frac{1}{2}$. Second, it follows from the properties of $\alpha(.)$ that $\frac{\partial \alpha(C_L)/\partial C}{\partial \alpha(C_R)/\partial C} \geq 1$. Finally, it follows from the properties of $q(.)$ that $\frac{\partial q(I^0 - C_L)/\partial I}{\partial q(I^0 - C_R)/\partial I} \leq 1$. Combining these three facts, it follows that $\frac{\partial \alpha(C_L)/\partial C}{\partial \alpha(C_R)/\partial C}\hat{\gamma} > \frac{\partial q(I^0 - C_L)/\partial I}{\partial q(I^0 - C_R)/\partial I}(1-\hat{\gamma})$, which based on Equation 3A implies that $\rho > 1 - \rho$. Therefore, party R will be better off by deviating to $C_R = C_L$, where $\rho = \frac{1}{2}$. Consequently, $C_L < C_R$ cannot happen at an interior equilibrium. Following the same steps, it can be also proved that $C_R < C_L$ never happen. To do so, some adjustment to our computations are required. Since the assumption that $C_R \leq C_L$ implies that $\hat{\gamma} \leq \frac{1}{2}$, Equations 6, 7, and 8 change: one must replace C_L by C_R and vise-versa in all these equations. Hence, $C_R = C_L$ at equilibrium. Using this fact, the conditions in Equations 1A and 2A reduce to Equation 14. The concavity of $\alpha(.)$ and $q(.)$ are not enough to guarantee that C^* represents a maximum. It can be proved that $\frac{\partial^2 B_k}{\partial C^2} < 0$ if $e \leq 4\bar{\alpha}$. From here, we get the restriction on the upper bound of e.

We now prove that this equilibrium exists and is unique. It can be verified that the left-hand side function in Equation 14 is a continuous strictly increasing function of C that goes from $\delta \lim_{I\to 1} \frac{\partial q}{\partial I} \in [0, \infty)$ (as C goes to zero) to infinite (as C goes to I^0). It can be also verified that the right-hand side function in Equation 14 is a continuous, strictly decreasing function of C, that goes from infinity (as C goes to zero) to $\frac{1}{2}\frac{a}{\alpha(I^0)}$ (as C goes to I^0), where $a > \lim_{C_k \to 1} \frac{\partial \alpha}{\partial C_k}$, and $\alpha(I^0) < \bar{\alpha}$. Therefore, the marginal cost and the marginal benefit of contributions always cross each other at some unique point $C^* \in (0, I^0)$, meaning that an interior equilibrium always exists. Furthermore, this is unique.

Let us now consider corner solutions. First, assume that both 10 and 11 are binding—i.e., both hold with equality—and that at equilibrium $C_L < C_R$. This last inequality holds if and only if $\frac{C_L}{\mu + C_L} < \frac{C_R}{\mu + C_R}$, which in turn implies that $\rho < 1 - \rho$. Thus, party L has incentive to increase C_L and consequently it cannot be an equilibrium. Following the same steps, and keeping in mind the change mentioned above on Equations 6, 7, and 8, it can be also proved that $C_R < C_L$ never happen. Hence, in this case, $C_L^* = C_R^*$. Using this result and the constraints in Equations 10 and 11 with equality, it follows that $C_L^* = C_R^* = \mu$. Due to symmetry, the case where only one contractor's constraint binds cannot be an equilibrium. If there exist a C_L^* such that 1A holds with equality, then $C_R = C_L^*$ also satisfies equation 2A and vise-versa.

Proof for Proposition 2.

a) We already know from the main text that, regardless of the value of $\alpha(C^*)$, a small reduction in contributions below C^* leaves unchanged the payoffs of parties and contractors 3 to n; and increase the payoffs of contractors 1 and 2, and always-abstainers. Assuming that $\alpha(C^*) < \frac{1}{2}$, we now analyze how this reduction affects the welfare of the rest of the citizens. Imposing this assumption and using Equation 14, it follows that $\frac{\partial \alpha(C^*)}{\partial C} < \delta \frac{\partial q(I^0 - C^*)}{\partial I}$. Consider first always-voters. The change in their payoffs where there is a small reduction in contributions below C^* is given by $\delta \frac{\partial q(I^0 - C^*)}{\partial I}$-$\frac{\partial \alpha(C^*)}{\partial C}$. The assumption on $\alpha(C^*)$ guarantees that this change is positive.

Let us now consider switchers. As there is a small reduction in contributions from C^* to C', switchers are also confronted with loses and gains. On the one hand, since $e\left|\frac{1}{2} - \gamma\right| - x + \alpha(C') < 0$ and $e\left|\frac{1}{2} - \gamma\right| - x + \alpha(C^*) \geq 0$, it follows that switchers' voting costs satisfy the following conditions: $e\left|\frac{1}{2} - \gamma\right| + \alpha(C') < x \leq e\left|\frac{1}{2} - \gamma\right| + \alpha(C^*)$. Consequently, as contributions go from C^* to C', the maximum payoff lost by a switcher is given by $\alpha(C^*) - \alpha(C')$, while the minimum payoff lost by a switcher is zero. On the other hand, switchers' gains as contributions are reduced are given by $\delta[q(I^0 - \alpha(C')) - q(I^0 - \alpha(C^*))]$. Comparing losses and gains, it follows that the total payoff for switchers increases as contributions are reduced if and only if $\frac{\partial \alpha(C^*)}{\partial C} < \delta \frac{\partial q(I^0 - C^*)}{\partial I}$. Hence, we conclude that contributions C^* are Pareto inefficient if $\alpha(C^*) < \frac{1}{2}$, and that a small reduction in contributions below C^* Pareto dominates unrestricted contributions.

b) As we already know from the main text, regardless of the value of $\alpha(C^*)$, there is no contribution $C \in (C^*, 1)$ that Pareto dominates C^*. Assume now that $\alpha(C^*) \geq \frac{1}{2}$, which from Equation 14 implies in turn that $\frac{\partial \alpha(C^*)}{\partial C} \geq \delta \frac{\partial q(I^0 - C^*)}{\partial I}$. Using this fact and the same arguments used in the proof for Proposition 2(a), it follows that a small reduction in contributions below C^* negatively affects the payoffs of always-voters and switchers. Hence, C^* is not Pareto dominated by any other contribution amount $C \in [0, 1]$ if $\alpha(C^*) \geq \frac{1}{2}$.

Proof for Proposition 3.

Suppose that $\bar{\alpha} < 1/2$.

a) We can anticipate from our discussion in section 4 that banning contributions is *welfare-improving* for always-abstainers (those who abstain from voting when $C = C^*$ and $C = 0$), and for contractors 3 to n—their expected payoff increases from zero to $\mu/n > 0$. The effect of banning contributions on the payoffs for contractors 1 and 2 is ambiguous, and depends not only on δ, but also on n—their payoffs go from $\frac{1}{2}(\mu - C^*)$ to μ/n.

Consider always-voters and switchers. The change in the payoff for always-voters as contributions are banned is given by $1-\delta q(I^0-C^*)-\alpha(C^*)$. Therefore, banning contributions is *welfare-improving* for always-voters if and only if $1 - \delta q(I^0 - C^*) > \alpha(C^*)$; otherwise, it is not. The change in the payoff for switcher as contributions are banned is given by $1 - \delta q(I^0 - C^*) - \alpha(C^*) + e\left|\frac{1}{2} - \gamma\right| - x$. Since they end up not voting in this case, it follows that $e\left|\frac{1}{2} - \gamma\right| - x < 0$. Hence, banning contributions is also *welfare-improving* for switchers if and only if $1-\delta q(I^0-C^*) > \alpha(C^*)$; otherwise, it is not. The first statement in Proposition 3(a) follows from the fact that $\bar{\alpha} > \alpha(C^*)$.

b) The properties of $q(.)$ and $\alpha(.)$ and the assumption that $\bar{\alpha} < 1/2$ ensure that there always exist a $C^E \in (0, C^*)$ such that $\frac{\partial\alpha(C^E)}{\partial C} = \delta\frac{\partial q(I^0-C^E)}{\partial I}$. Combining this with the result in Proposition 2(a), it follows that reducing contributions from C^* to any $\bar{C} \in [C^E, C^*)$ Pareto dominates unrestricted contributions. Furthermore, C^E Pareto dominates any other limit in the interval $[C^E, C^*)$.

Proof for Proposition 4.

Suppose that $\bar{\alpha} < 1/2$.

a) Consider funding campaigns with public resources $\beta \in (0, C^*)$, while simultaneously banning contributions. We already know that banning contributions is *welfare-improving* for contractors 3 through n, and has an ambiguous effect on the payoffs for contractors 1 and 2. Furthermore, subsidizing campaigns while simultaneously banning contributions is *welfare-improving* for always-abstainers if and only if $\beta < C^*$.

Computing the change in the payoffs for ways-voters as we move from unrestricted campaign contributions to this policy, it follows that it is *welfare-improving* for them if and only if $\delta[q(I^0-\beta)-q(I^0-C^*)] > \alpha(C^*)-\alpha(\beta/2)$; otherwise, it is not. Using the same arguments used in the proof for Proposition 3(a), it can be shown that this policy is also *welfare-improving* for switchers if this inequality holds; otherwise, it is not. The first statement in Proposition 4(a) follows from the fact that $\bar{\alpha} > \alpha(C^*)$.

Whether or not there exist at least one $\beta < C^*$ such that $\bar{\alpha} < \delta[q(I^0 - \beta) - q(I^0 - C^*)] + \alpha(\beta/2)$, critically depends on the specific form of both, $\alpha(.)$ and $q(.)$. The value of β that maximizes the payoff for always-voters under this policy, β^*, is implicitly given by $\delta\frac{\partial q(I^0-\beta^*)}{\partial I} = \frac{1}{2}\frac{\partial\alpha(\beta^*/2)}{\partial C}$. The condition $\frac{1}{2}\frac{\partial\alpha(C/2)}{\partial C} < \frac{\partial\alpha(C)}{\partial C}$ guarantees that $\beta^* < C^*$. If this happens, then $\frac{1}{2}\frac{\partial\alpha(C^*/2)}{\partial C} < \frac{1}{2\alpha(C^*)}\frac{\partial\alpha(C^*)}{\partial C}$. Using the equilibrium condition in Equation 14, it follows that $\frac{1}{2}\frac{\partial\alpha(C^*/2)}{\partial C} < \delta\frac{\partial q(I^0-C^*)}{\partial I}$. Since β^* equalizes these two terms, it must be the case that $\beta^* < C^*$. This guarantees that implementing β^* is *welfare-improving* for always-abstainers. Furthermore, at least β^* is *welfare-improving* for always-voters and switchers if $\bar{\alpha} < \delta[q(I^0 - \beta^*) - q(I^0 - C^*)] + \alpha(\beta^*/2)$.

b) Consider funding campaigns with public resources $\tilde{\beta} \in (0, 1]$, while simultaneously imposing a contribution limit $\tilde{C} \in (0, C^*)$, with $\tilde{\beta} + \tilde{C} < C^*$. We already know that limiting contributions is *welfare-improving* for contractors 1 and 2, and does not affect the payoffs of contractors 3 through n. Subsidizing campaigns while simultaneously limiting contributions is *welfare-improving* for always-abstainers if and only if $\tilde{C} + \beta < C^*$.

Computing the change in the always-voters' payoffs as we move away from unrestricted campaign contributions towards this policy, it follows that doing so is *welfare-improving* for them if and only if $\delta[q(I^0 - \tilde{\beta} - \tilde{C}) - q(I^0 - C^*)] > \alpha(C^*) - \alpha(\tilde{C} + \tilde{\beta}/2)$; otherwise, it is not. Using the same arguments used in the proof for Proposition 3(a), it can be shown that this policy is also *welfare-improving* for switchers if this inequality holds; otherwise, it is not. The first statement in Proposition 4(b) follows from the fact that $\bar{\alpha} > \alpha(C^*)$.

Let us now assume that subsidizing campaigns with β^*—defined as in Proposition 4(a)—while simultaneously banning contributions is *welfare-improving* for all groups of citizens. Take $\tilde{C} + \tilde{\beta} = \beta^* < C^*$. It then follows that $q(I^0 - \tilde{C} - \tilde{\beta}) + \alpha(\frac{\tilde{C}+\tilde{\beta}}{2}) > \delta q(I^0 - C^*) + \alpha(C^*)$. Since $\alpha(\frac{\tilde{C}+\tilde{\beta}}{2}) < \alpha(\tilde{C} + \frac{\tilde{\beta}}{2})$, and $\bar{\alpha} > \alpha(C^*)$, it follows that $\bar{\alpha} < \delta[q(I^0 - \tilde{C} - \tilde{\beta}) - q(I^0 - C^*)] + \alpha(\tilde{C} + \tilde{\beta}/2)$. This guarantees that $(\tilde{C}, \tilde{\beta})$, with $\tilde{C} + \tilde{\beta} = \beta^*$ is Pareto improving. Furthermore, there exists an infinite number of policies $(\tilde{C}, \tilde{\beta})$ that do the same job.

Proof for Proposition 5.

Suppose that $\bar{\alpha} < \frac{1}{2}$.

a) Let us allow for unrestricted contributions, impose a lump-sum tax $\tau \in (0, \bar{\tau})$ on contributions, and invest τ in the public-works project. $\bar{\tau}$—defined below—is the maximum rate where the unrestricted contributions at equilibrium are interior. Consider imposing the tax on parties. The voting subsidy is then given by $\alpha(C_k - \tau)$, and the quality of the public-works project is given by $\delta q(I^0 - C_k + 2\tau)$. Following the same steps used to compute the optimal unrestricted contributions without taxes, it follows that the contributions with taxes at equilibrium are such that $C_k^\tau = C^\tau$, and are implicitly given by:

$$\delta \frac{\partial q\left(I^0 - C^\tau + 2\tau\right)}{\partial I} = \frac{1}{2\alpha\left(C^\tau - \tau\right)} \frac{\partial \alpha\left(C^\tau - \tau\right)}{\partial C} \tag{5A}$$

Following the same arguments used in the Proof for Proposition 1, one can prove that this equilibrium always exists and is unique. Using the implicit function theorem in Equation 5A, it follows that $\partial C^\tau / \partial \tau > 0$. Hence, the maximum τ that can be imposed to obtain an interior solution is implicitly defined by replacing C^τ by μ in Equation 5A. Moreover, increasing τ beyond $\bar{\tau}$ leaves unrestricted contributions with taxes unchanged at μ.

We now prove that $C^\tau - 2\tau < C^* < C^\tau - \tau$. If this is true, then $\alpha(C^\tau - \tau) > \alpha(C^*)$, and $q(I^0 - C^\tau + 2\tau) > q(I^0 - C^*)$. To do this, we use the following two facts. *Fact 1:* $\alpha(C^*) = \frac{1}{2\delta} \frac{\partial \alpha(C^*)/\partial C}{\partial q(I^0 - C^*)/\partial I}$, from the equilibrium condition in Equation 14. *Fact 2:* $\alpha(C^\tau - \tau) = \frac{1}{2\delta} \frac{\partial \alpha(C^\tau - \tau)/\partial C}{\partial q(I^0 - C^\tau + 2\tau)/\partial I}$, from the equilibrium condition in Equation 5A. We first prove that $C^* < C^\tau$. Assume it is not, i.e., $C^* \geq C^\tau$. This assumption implies both that $C^* > C^\tau - \tau$ and $I^0 - C^* < I^0 - C^\tau + 2\tau$. It follows from these two inequalities that $\frac{\partial \alpha(C^*)}{\partial C} < \frac{\partial \alpha(C^\tau - \tau)}{\partial C}$, and $\frac{\partial q(I^0 - C^*)}{\partial I} > \frac{\partial q(I^0 - C^\tau + 2\tau)}{\partial I}$. Combining these last two inequalities, and using *Fact 1* and *2*, it follows that $\alpha(C^*) < \alpha(C^\tau - \tau)$. Nevertheless, from the initial assumption, we know that $\alpha(C^*) > \alpha(C^\tau - \tau)$, which is a contradiction. Consequently, $C^* < C^\tau$.

We now prove that $C^* < C^\tau - \tau$. Assume that it is not, i.e., $C^* \geq C^\tau - \tau$. From here, we can use the same steps used in the previous paragraph and prove that this implies that

$C^* < C^\tau - \tau$, which contradicts the initial assumption. Finally, we prove that $C^\tau - 2\tau < C^*$. Assume that it is not, i.e., $C^\tau - 2\tau \geq C^*$. It implies that $\frac{\partial q(I^0 - C^\tau + 2\tau)}{\partial I} \geq \frac{\partial q(I^0 - C^*)}{\partial I}$. Moreover, since we already know that $C^* < C^\tau - \tau$, it is true that $\frac{\partial \alpha(C^\tau - \tau)}{\partial C} < \frac{\partial \alpha(C^*)}{\partial C}$. Combining these two inequalities and using *Fact 1* and *Fact 2*, it follows that $\alpha(C^\tau - \tau) < \alpha(C^*)$. Nevertheless, we know form the initial assumption that $\alpha(C^\tau - 2\tau) \geq \alpha(C^*)$, which is a contradiction. Consequently, $C^\tau - 2\tau < C^*$.

Consider now that the tax is imposed on contractors 1 and 2. As said in the main text, the voting subsidy is given by $\alpha(C_k)$, while the quality of the public-works project is given by $\delta q(I^0 - C_k + \tau)$. It can be proved that the equilibrium is symmetric, always exist and is unique. Contributions at equilibrium are now implicitly given by:

$$\delta \frac{\partial q\left(I^0 - C^\tau + \tau\right)}{\partial I} = \frac{1}{2\alpha\left(C^\tau\right)} \frac{\partial \alpha\left(C^\tau\right)}{\partial C} \tag{6A}$$

The maximum τ that can be imposed to obtain an interior solution is implicitly defined by replacing C^τ by $\mu - \tau$ in Equation 6A. Using the same strategy used in the case where the tax was imposed on parties, it can be proved first that $C^\tau - \tau < C^*$, and before that $C^\tau > C^*$.

b) Consider simultaneously imposing a campaign contribution limit $\hat{C} \in (0, C^*)$; a contribution lump-sum tax $\hat{\tau} \in (0, \bar{\tau})$, and then investing the resulting tax revenues on the public-works project. The voting subsidy and the quality of the project are now given by $\alpha(\hat{C} - \hat{\tau})$, and $q(I^0 - \hat{C} + 2\hat{\tau})$, as the tax is imposed on parties. Consider any policy $(\hat{C}, \hat{\tau})$ with $\hat{C} - \hat{\tau} = C^E < C^*$, where C^E is defined as in Proposition 3. Consequently, $\hat{C} - 2\hat{\tau} = C^E - \hat{\tau}$. It then follows that $\alpha(\hat{C} - \hat{\tau}) + q(I^0 - \hat{C} + 2\hat{\tau}) > \alpha(C^E) + q(I^0 - C^E)$. Since we already know from proposition 3(b) that $\alpha(C^E) + q(I^0 - C^E) > \alpha(C^*) + q(I^0 - C^*)$, this policy is not only *welfare-improving* for all groups of citizens, but also gives them a greater payoff than the one obtained when just limiting contributions. Furthermore, this policy is *welfare-improving* for contractors 1 and 2, and does not affect the payoffs of the other contractors. Notably, there exists an infinite number of policies $(\hat{C}, \hat{\tau})$ that satisfy $\hat{C} - \hat{\tau} = C^E$. The same can be proved if the tax is imposed on contractors 1 and 2, but with $\hat{C} = C^E$.